Date: 2010-05-11

Final Exam: Chapters 5-9, 11
Mth 164-280

Name: _____

General Test Instructions:

You may use your calculator or any software on your computer to directly calculate the answers. **All answers must be recorded on a Scantron form.**

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1. Find the length (to the nearest hundredth of an inch) of an arc that subtends a central angle 105° in a circle of radius 12 inches.

    a. 21.99 inches
    b. 22.06 inches
    c. 21.78 inches
    d. 24.68 inches
    e. None of these

2. A wheel is rotating a 8 revolutions per second. Find its angular speed in radians per second (to the nearest hundredth).

    a. 50.14 radians per second
    b. 51.98 radians per second
    c. 49.96 radians per second
    d. 50.27 radians per second
    e. None of these

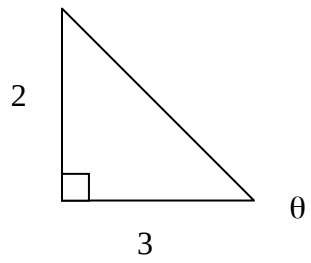
3. If θ is an acute angle such that $\tan\theta = \frac{5}{9}$, find the exact value of $\csc\theta$.

   a. 2
   b. $\frac{2\sqrt{26}}{5}$
   c. $\frac{\sqrt{106}}{5}$
   d. $\frac{2\sqrt{29}}{7}$
   e. None of these

4. Use a calculator to find the value of $\csc 36°$ to the nearest ten-thousandth.

   a. −7.8147
   b. 1.7013
   c. 1.3656
   d. 1.2361
   e. None of these

5. Find the values of $\sin\theta, \cos\theta, \text{and } \tan\theta$ for the right triangle shown below.



   a. $\sin\theta = \frac{3\sqrt{13}}{13}, \cos\theta = \frac{2\sqrt{13}}{13}, \tan\theta = \frac{3}{2}$
   b. $\sin\theta = \frac{2\sqrt{13}}{13}, \cos\theta = \frac{3\sqrt{13}}{13}, \tan\theta = \frac{2}{3}$
   c. $\sin\theta = \frac{3\sqrt{5}}{5}, \cos\theta = \frac{2\sqrt{5}}{5}, \tan\theta = \frac{3}{2}$
   d. $\sin\theta = \frac{2\sqrt{5}}{5}, \cos\theta = \frac{3\sqrt{5}}{5}, \tan\theta = \frac{3}{2}$
   e. None of these

6. Use the Reference Angle Theorem to find the exact value of $\sec\left(-\frac{3\pi}{4}\right)$

   a. $\sqrt{2}$
   b. $-\sqrt{2}$
   c. $\frac{\sqrt{2}}{2}$
   d. $-\frac{\sqrt{2}}{2}$
   e. None of these

7. Express $\frac{\csc^2\theta - 1}{\csc^2\theta}$ as a single term that involves the cosine function.

   a. $cos^2\theta$
   b. $2\cos\theta$
   c. $\frac{1}{cos^2\theta}$
   d. $cos^2\theta - 1$
   e. None of these

8. Fine the amplitude, period, and phase shift for the function $= -2\sin\left(3x - \frac{\pi}{6}\right)$.

   a. $2, \frac{2\pi}{3}, \frac{\pi}{6}$
   b. $-2, \frac{2\pi}{3}, \frac{\pi}{18}$
   c. $2, \frac{2\pi}{3}, \frac{\pi}{18}$
   d. $-2, \frac{2\pi}{3}, \frac{\pi}{6}$
   e. None of these

9. Simplify the expression $\frac{sin^2x\cos x}{cos^2x} + \cos x$ to produce one of the following:

   a. $\sec x$
   b. cos^2x
   c. $\cos x$
   d. $\csc x$

10. Simplify the expression $\frac{\sin x}{1+\sin x} - \frac{\sin x}{1-\sin x}$ to produce one of the following:
    a. $-2sin^2x$
    b. -2
    c. sec^2x
    d. $-2tan^2x$

11. Given $\cos\alpha = -7/25$, with α in Quadrant III, and $\cos\beta = 8/17$, with β in Quadrant I, find the *exact* value of $\cos(\alpha - \beta)$.

    a. $-\frac{304}{425}$
    b. $-\frac{416}{425}$
    c. $\frac{304}{425}$
    d. $-\frac{56}{425}$
    e. None of these

12. Write $cos^2 4\theta - sin^2 4\theta$ as a single term:

    a. $\cos 4\theta$
    b. 1
    c. $\sin 8\theta$
    d. $\cos 8\theta$
    e. None of these

13. Find the exact value of $\sin\frac{\alpha}{2}$, given that $\cos\alpha = -7/24$, with α in Quadrant III.
    a. $\frac{\sqrt{93}}{12}$
    b. $\frac{\sqrt{51}}{12}$
    c. $-\frac{\sqrt{93}}{12}$
    d. $-\frac{\sqrt{51}}{12}$
    e. None of these

14. Find the exact value of $\cos 2\alpha$, given that $\cos \alpha = -7/12$, with α in Quadrant II.

a. $-\frac{31}{18}$
b. $\frac{31}{18}$
c. $-\frac{23}{72}$
d. $\frac{23}{72}$
e. None of these

15. Write the expression $2cos^2 4\theta - 1$ as a single term

a. $\cos 2\theta$
b. $\cos 8\theta$
c. 1
d. $\sin 8\theta$
e. None of these

16. Find the *exact* value of $\sin\left(\cos^{-1}\frac{7}{25}\right)$

a. $\frac{25}{7}$
b. $\frac{25}{24}$
c. $\frac{7}{24}$
d. $\frac{24}{25}$
e. None of these

17. Solve $2\sin^2 x + 9\cos x + 9 = 0$ where $0 \leq x < 2\pi$

a. $\frac{3\pi}{2}$
b. π
c. $0, \frac{\pi}{2}$
d. 0
e. None of these

18. Solve the triangle ABC, with angle $A = 50°$, angle $B = 81°$, and side $c = 12$ miles. Round sides *a* and *b* to the nearest hundredth of a mile.

    a. $C = 49°,\ a = 15.70$ miles, $b = 12.80$ miles
    b. $C = 49°,\ a = 12.18$ miles, $b = 15.70$ miles
    c. $C = 49°,\ a = 3.30$ miles, $b = 7.93$ miles
    d. $C = 49°,\ a = 7.93$ miles, $b = 3.30$ miles
    e. None of these

19. In triangle ABC, angle $B = 44°$, side $a = 29$ and side $c = 17$. Find side *b* (round to the nearest whole unit).

    a. 39
    b. 420
    c. 150
    d. 21
    e. None of these

20. In triangle ABC, side $a = 20$, side $b = 33$, and side $c = 18$. Find angle A (to the nearest tenth of a degree).
    a. 148.5°
    b. 11.8°
    c. 42.3°
    d. 31.5°
    e. None of these

21. Given angle $A = 34°$, angle $B = 58°$, and side $a = 6$ units, find the area of triangle ABC. Round to the nearest tenth of a square unit.

    a. 2.8 units^2
    b. 15.3 units^2
    c. 27.3 units^2
    d. 54.6 units^2
    e. None of these

22. Use Heron's formula to find the area (to the nearest square inch) of a triangle with sides of length 23 inches, 29 inches, and 12 inches.

    a. $262\ \text{in}^2$
    b. $131\ \text{in}^2$
    c. $78\ \text{in}^2$
    d. $147\ \text{in}^2$
    e. None of these

23. A vector has a magnitude of 15 and a direction of 230°. Write the vector in the form $a_1\mathbf{i} + a_2\mathbf{j}$. State a_1 and a_2 rounded to the nearest hundredth.

    a. $-7.36\mathbf{i} - 12.42\mathbf{j}$
    b. $-9.64\mathbf{i} + 11.49\mathbf{j}$
    c. $-9.64\mathbf{i} - 11.49\mathbf{j}$
    d. $-8.24\mathbf{i} - 10.48\mathbf{j}$
    e. None of these

24. Given $\mathbf{u} = 5\mathbf{i} + 10\mathbf{j}\ and\ \mathbf{v} = \mathbf{i} - 3\mathbf{j}$, find $2\mathbf{u} - 9\mathbf{v}$.
    a. $19\mathbf{i} - 7\mathbf{j}$
    b. $\mathbf{i} + 47\mathbf{j}$
    c. $\mathbf{i} - 7\mathbf{j}$
    d. $-80\mathbf{i} + 47\mathbf{j}$
    e. None of these

25. Find the dot product of $\mathbf{u} = 2\mathbf{i} - 7\mathbf{j}\ and\ \mathbf{v} = \mathbf{i} + 8\mathbf{j}$.
    a. -54
    b. -13
    c. 58
    d. -12
    e. None of these

26. A triangular piece of property costs \$5.80 per square foot. If the lot measures 90 feet by 80 feet by 70 feet, find the cost of the property. Round to the nearest hundred dollars.

    a. \$2,700
    b. \$308,100
    c. \$15,600
    d. \$20,900
    e. None of these

27. Find the vertex, focus, and directrix of the parabola given by $2y^2 = -3x$.

a. Vertex: $(0,0)$; focus: $\left(\frac{3}{8}, 0\right)$; directrix: $x = -\frac{3}{8}$
b. Vertex: $(0,0)$; focus: $\left(-\frac{3}{8}, 0\right)$; directrix: $x = \frac{3}{8}$
c. Vertex: $(0,0)$; focus: $(3,0)$; directrix: $x = -3$
d. Vertex: $(0,0)$; focus: $(-3,0)$; directrix: $x = 3$
e. None of these

28. Find the vertex, focus, and directrix of the parabola given by $x^2 + 8x - 16y + 80 = 0$.

a. Vertex: $(4, -4)$; focus: $(4,0)$; directrix: $y = -8$
b. Vertex: $(-4, 4)$; focus: $(-4, 20)$; directrix: $y = -12$
c. Vertex: $(4, -4)$; focus: $(8, -4)$; directrix: $x = 0$
d. Vertex: $(-4, 4)$; focus: $(-4, 8)$; directrix: $y = 0$
e. None of these

29. Find the vertices and foci of the ellipse given by $5x^2 + y^2 + 60x + 4y + 159 = 0$.

a. Vertices: $(-11, -2)$ and $(-1, -2)$; foci: $\left(-6 + 2\sqrt{5}, -2\right)$ and $\left(-6 - 2\sqrt{5}, -2\right)$
b. Vertices: $(6, 7)$ and $(6, -3)$; foci: $\left(6, 2 + 2\sqrt{5}\right)$ and $\left(6, 2 - 2\sqrt{5}\right)$
c. Vertices: $(6, 7)$ and $(6, -3)$; foci: $\left(6 + 2\sqrt{5}, 2\right)$ and $\left(6 - 2\sqrt{5}, 2\right)$
d. Vertices: $(-6, 3)$ and $(-6, -7)$; foci: $\left(-6, -2 + 2\sqrt{5}\right)$ and $\left(-6, -2 - 2\sqrt{5}\right)$
e. None of these

30. Find the equation, in standard form, of the ellipse that has foci at $\left(-3 + \sqrt{15}, 6\right)$ and $\left(-3 - \sqrt{15}, 6\right)$, and the length of whose minor axis is 12.

a. $\frac{(x+3)^2}{159} + \frac{(y-6)^2}{144} = 1$
b. $\frac{(x-3)^2}{39} + \frac{(y+6)^2}{24} = 1$
c. $\frac{(x+3)^2}{51} + \frac{(y-6)^2}{36} = 1$
d. $\frac{(x-3)^2}{39} + \frac{(y-6)^2}{24} = 1$
e. None of these

31. Find the vertices and asymptotes of the hyperbola given by $\frac{y^2}{100} - \frac{x^2}{81} = 1$.

a. Vertices: $(0, 10)$ and $(0, -10)$; asymptotes: $y = \pm\frac{9}{10}x$
b. Vertices: $(9, 0)$ and $(-9, 0)$; asymptotes: $y = \pm\frac{9}{10}x$
c. Vertices: $(9, 0)$ and $(-9, 0)$; asymptotes: $y = \pm\frac{10}{9}x$
d. Vertices: $(0, 10)$ and $(0, -10)$; asymptotes: $y = \pm\frac{10}{9}x$
e. None of these

32. Which of the following statements describes the graph of $x^2 - 4x + y^2 = 5$?

a. It is a parabola that opens up.
b. It is a hyperbola that opens up and down.
c. It is an ellipse.
d. It is a hyperbola that opens left and right.
e. None of these

33. Determine which of the following equations can be produced by eliminating the parameter *t* from $= t + 5, y = t^2 + 3$.

a. $y = x^2 - 22$
b. $y = x^2 - 28$
c. $y = x^2 - 10x + 28$
d. $y = x^2 + 10x + 28$
e. None of these

34. Solve the system: $\begin{cases} 2x - 3y = 15 \\ y = \frac{2}{3}x - 10 \end{cases}$ What is the value of *x* in the solution?

a. $x = -\frac{15}{4}$
b. $x = \frac{15}{4}$
c. $x = \frac{45}{4}$
d. The system is inconsistent, there is no solution.

35. Solve the system: $\begin{cases} x+5y-z=22 \\ 3x+4y-4z=10 \\ x+y-z=2 \end{cases}$ What is the value of z in the solution?

a. $z=1$
b. $z=-2$
c. $z=-3$
d. The system is inconsistent, there is no solution.
e. None of these

36. Solve the system: $\begin{cases} x+y-4z=6 \\ 2x-5y+3x=2 \\ x-6y+7z=-4 \end{cases}$ What is the value of y in the solution?

a. $y=2$
b. $y=3$
c. $y=5$
d. The system is inconsistent, there is no solution.
e. None of these

37. Solve the dependent system: $\begin{cases} x-y+z=-6 \\ 2x-3y+4y=-12 \end{cases}$ Let $z=c$ in your solution

a. $(2c, c+6, c)$
b. $(2c, c-6, c)$
c. $(c-6, 2c, c)$
d. $(c+6, c, c)$
e. None of these

38. Solve the system: $\begin{cases} y=2x+2 \\ y=x^2+4x-1 \end{cases}$ What is the value of x in the solution?

a. $x=1$
b. $x=1$ and $x=-3$
c. $x=-1$ and $x=3$
d. There is no real solution, the graphs never intersect.
e. None of these

39. Solve the system: $\begin{cases} x^2 - 100y^2 = 100 \\ x^2 + y^2 = 100 \end{cases}$ What is the value of *x* in the solution?

a. $x = -10$ and $x = 10$
b. $x = 10$
c. $x = 0$
d. There is no real solution, the graphs never intersect.
e. None of these

40. Find the equation of the circle that passes through the points $(4, 10), (1, 1),$ and $(8, 2)$.

a. $x^2 + 4x + y^2 - 5y + 25 = 0$
b. $x^2 - 8x + y^2 - 10y + 16 = 0$
c. $x^2 - 8x + y^2 - 5y + 25 = 0$
d. $x^2 + 4x + y^2 - 10y + 16 = 0$
e. None of these

41. Flying with the wind, a plane traveled 312 miles in 2 hours. Flying against the wind, the plane traveled the same distance in 3 hours. Find the rate of the plane in calm air and the rate of the wind.

a. plane: 156 mph; wind 26 mph
b. plane: 150 mph; wind 6 mph
c. plane: 130 mph; wind 26 mph
d. plane: 120 mph; wind 36 mph
e. None of these

42. A silversmith has two alloys. The first alloy is 28% silver, and the second is 60% silver. How many grams of each should be mixed to produce 80 grams of an alloy that is 52% silver?

a. 15 grams of 28% silver, 65 grams of 60% silver
b. 20 grams of 28% silver, 60 grams of 60% silver
c. 25 grams of 28% silver, 55 grams of 60% silver
d. 30 grams of 28% silver, 50 grams of 60% silver
e. None of these

43. Find the fifth (5^{th}) term of the sequence that has an n^{th} term of $\frac{(-1)^n(n+2)!}{4}$

a. 1260
b. 1.75
c. −1260
d. −1.75
e. None of these

44. Find the fourth (4^{th}) term of the sequence defined by the recursive formula

$$a_1 = -5,\ a_2 = 3;\ \ldots\ a_n = na_{n-1} + 5a_{n-2} \text{ for } n \geq 3$$

a. −49
b. −16
c. −325
d. 16
e. None of these

45. The sequence defined by $a_n = 15 - 4n$ is:

a. an arithmetic sequence
b. a geometric sequence
c. the Fibonacci sequence
d. the binomial sequence
e. None of these

46. The sequence defined by $a_n = \frac{1}{4}\left(-\frac{2}{3}\right)^{n-1}$ is:

a. an arithmetic sequence
b. a geometric sequence
c. the Fibonacci sequence
d. the binomial sequence
e. None of these

47. The sequence defined by $a_n = \frac{10}{n!}$ is:

a. an arithmetic sequence
b. a geometric sequence
c. the Fibonacci sequence
d. the binomial sequence
e. None of these

48. Write $-5 + 6 - 7 + 8 - 9 + 10$ in summation notation.

a. $\sum_{k=1}^{6}(-1)^k(4k+1)$
b. $\sum_{k=0}^{5}(-1)(4k+1)$
c. $\sum_{k=1}^{6}(-1)^k(4k-1)$
d. $\sum_{k=1}^{6}(-1)^k(k+4)$
e. None of these

49. Find: $\sum_{k=2}^{7}(-1)^k(3k-2)$

a. -15
b. -9
c. -6
d. 15
e. None of these

50. Write $0.2\overline{61}$ as the quotient of two integers. Give your answer in lowest terms.

a. $\frac{261}{99}$
b. $\frac{259}{999}$
c. $\frac{259}{990}$
d. $\frac{289}{980}$
e. None of these